CALCULATION OF THE COMPRESSION OF A PLASMA BY A CYLINDRICAL LINER WITH A MAGNETIC FIELD

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Different thermonuclear liner systems using a magnetic field as thermal insulation have been studied intensively in the last few years [1-4]. The main idea involved in thermonuclear liner systems is that the plasma generated beforehand in the liner is compressed by the cumulating liner and is heated up to a thermonuclear temperature. The liner can be accelerated by different methods.

In this work the process of compression with a cylindrical, cumulating, conducting, incompressible liner of a plasma cylinder, arranged coaxially in the liner and insulated from the liner initially by an axial magnetic field, is studied for a cylindrical one-dimensional geometry. A simplified mathematical model of the process of shock-free compression of a one-temperature ideally conducting plasma, consisting of a DT mixture, taking into account the physical factors (release of energy from thermonuclear burn, bremstrahlung losses, penetration of the magnetic field into the liner taking into account Joule heating, and retardation of the liner) that affect the character of the compression, is constructed and studied numerically (under the assumption that the starting temperature of the plasma is so high that the velocity of sound in the plasma is much greater than the velocity of the liner). The mathematical model of the process of compression of the plasma by the liner studied in this work under the assumption that the liner is ideally conducting is identical to the mathematical model of [4].

The main purpose of this work is to compare the results of calculations based on these models, i.e., to study the effect of the diffusion of the magnetic field in the liner on the characteristics of the compression process.

We shall assume that that liner is a shell whose inner boundary has a given starting velocity v_0, and we shall assume that the electric conductivity of the liner is given by the law $\sigma = \sigma_0/(1 + \beta Q)$ (σ_0 is the conductivity of the liner at 0°C, β is a temperature factor, and Q is the increment to the heat content relative to the state at 0°C).

Neglecting the displacement current compared with the conduction current and the thermal conductivity, we shall write the equation, the initial condition, and the boundary conditions describing the process of penetration of the magnetic field into the liner in the Lagrangian system of coordinates (r, t) in the form (in the MKSA system of units)

$$\frac{\partial H}{\partial t} = \frac{1}{\sigma_0 \mu_0 r} \frac{\partial}{\partial r}\left[\frac{x^2}{r} \exp\left(\frac{\beta}{\sigma_0}\int_0^t \left(\frac{x}{r}\frac{\partial H}{\partial r}\right)^2 d\tau\right)\frac{\partial H}{\partial r}\right] \quad (R_{20} < r < R_{30},\ 0 < t),$$

$$H(r, 0) = H_0 + \frac{R_{20} - r}{R_{20} - R_{30}}(H_1 - H_0),$$

$$\left.\frac{\partial H}{\partial t}\right|_{r=R_{20}} [x_2^2(t) - x_1^2(t)] + H|_{r=R_{20}} \frac{d}{dt}[x_2^2(t) - x_1^2(t)] -$$
$$- \frac{2x_2^2(t)}{\sigma_0 \mu_0 R_{20}}\left.\left(\exp\left[\frac{\beta}{\sigma_0}\int_0^t \left(\frac{x}{r}\frac{\partial H}{\partial r}\right)^2 d\tau\right]\frac{\partial H}{\partial r}\right)\right|_{r=R_{20}} = 0, \quad H(r, t)|_{r=R_{30}} = H_1, \tag{1}$$

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where $\mu_0 = 4\pi\cdot10^{-7}$ V·sec/(A·m); R_{20} and R_{30} are the inner and outer starting radii of the liner; $x = x(r, t) = \sqrt{r^2 - R_{20}^2 + x_2^2(t)}$, and $t' = t$ are Euler coordinates; $x_2 = x_2(t)$ is the inner radius of the liner; $x_1 = x_1(t)$ is the radius of the plasma; H_0 is the starting magnetic field in the gap between the plasma and the liner; and, H_1 is the magnetic field at the outer boundary of the liner.

We denote $f(t) = H(r, t)|_{r-R_{20}} = H(x_2(t), t)$. From the equation of motion of the liner, taking into account the equation of continuity in order to find the boundary of the liner $x_2(t)$ we have

$$\frac{d^2x_2}{dt^2} = \frac{1}{x_2 \ln\frac{x_3}{x_2}}\left\{\left(\frac{dx_2}{dt}\right)^2\left[\frac{1}{2}\left(1-\frac{x_2^2}{x_3^2}\right) - \ln\frac{x_3}{x_2}\right] + \frac{\mu_0\left(f^2 - H_1^2\right)}{2\rho_+}\right\}, \tag{2}$$
$$x_2(0) = R_{20},\ dx_2/dt|_{t=0} = -v_0$$

($x_3 = x_3(t) = \sqrt{R_{30}^2 - R_{20}^2 + x_2^2(t)}$ is the outer radius of the liner and ρ_+ is the density of the liner material).

We shall assume that the compression of the plasma is uniform, i.e., the density does not depend on the spatial coordinate. Then $\rho = \rho_0 R_{10}^2/x_1^2(t)$ ($\rho = \rho(t)$ is the plasma density and $\rho_0 = \rho(0)$). We shall assume that the pressure in the plasma is always equal to the magnetic pressure $p = \mu_0 f^2(t)/2$, while the plasma is an ideal gas with the equation of state $p = 2(\rho k/m_H A)T$ ($m_H = 1.672\cdot10^{-27}$ kg, $k = 1.38\cdot10^{-23}$ J/K, $A = 2.5$, $T = T(t)$ is the temperature of the plasma). From the energy equation for the plasma taking into account the condition $x_1(0) = R_{10}$ (R_{10} is the initial radius of the plasma) we find the boundary of the plasma $x_1(t)$ in the form

$$\frac{dx_1}{dt} = \left[q(t) - \varepsilon(t) - \frac{3}{2}\mu_0 f\frac{df}{dt}\right]\frac{2x_1}{5\mu_0 f^2},\quad x_1(0) = R_{10}. \tag{3}$$

Here $\varepsilon(t)$ (J/(sec·m³)) are the bremmstrahlung losses; $q(t)$ (J/(sec·m³)) is the power liberated per unit volume owing to thermonuclear reactions in the DT mixture.

We shall assume that only reactions of the type ${}_1D^2 + {}_1T^3 \rightarrow {}_2He^4$ (3.6 MeV) + ${}_0n^1$ (14.1 MeV) occur, and that α particles release energy locally at the location where they are created. If the plasma temperature is introduced in the form

$$\theta(t) = \frac{\mu_0 f^2(t)\, m_H A x_1^2(t)}{1.16\cdot10^7 \rho_0 R_{10}^2 4k}\ (\text{keV}), \tag{4}$$

then it is well known [5] that

$$\varepsilon(t) = 0.49\cdot10^{-36}\left[\frac{\rho_0 R_{10}^2}{m_H A}\frac{1}{x_1^2(t)}\right]^2\sqrt{\theta(t)},$$
$$q(t) = \begin{cases} 11.19\cdot10^5\dfrac{\varepsilon(t)}{\theta^{7/6}(t)}\exp\left[-\dfrac{19.02}{\theta^{1/3}(t)}\right] & \text{for}\ \theta(t) < 10, \\ 10.04\cdot10^3\dfrac{\varepsilon(t)}{\theta^{7/6}(t)}\exp\left[-\dfrac{27.217}{\theta^{2/3}(t)} + \dfrac{3.638}{\theta^{1/3}(t)}\right] & \text{for}\ \theta(t) \geqslant 10. \end{cases} \tag{5}$$

The mathematical model (1)-(5) is a simplified model for the process of compression of the plasma by a cylindrical liner with a magnetic field studied in this work.

The problem (1)-(5) was solved by numerical methods for a wide range of values of the parameters. Figure 1 shows some results of the calculation for a copper liner with $\sigma_0 = 63.3\cdot10^6$ 1/(Ω·m), $\beta = 1.32\cdot10^{-9}$ m³/J, $H_0 = H_1 = 13.4\cdot10^6$ A/m, $R_{10} = 0.091$ m, $R_{20} = 0.11$ m, $R_{30} = 0.118$ m, $v_0 = 3\cdot10^3$ m/sec, $\rho_+ = 8.9\cdot10^3$ kg/m³, $\rho_0 = 0.45\cdot10^{-2}$ kg/m³. The curves 1-5 correspond to $x_1(t)$ (cm), $x_2(t)$ (cm), $f(t)$ (A/m), $\theta(t)$ (keV) and $W(t) = R_{20}\pi\int_0^t x_1^2(\tau)q(\tau)d\tau$ (MJ) is the energy released in the plasma over a distance equal to the radius of the liner in the vicinity of the maximum compression. Here, for comparison, the function W(t) (broken line) calculated

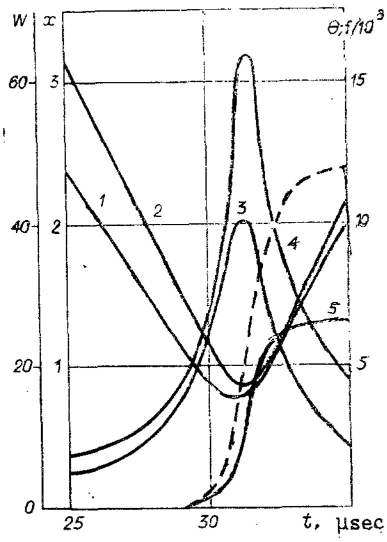

Fig. 1

under the assumption that there is no diffusion of the magnetic field in the liner [4] is also presented.

The calculations showed that taking into account the diffusion of the magnetic field in the liner with a reasonable coefficient of compression $R_{10}/x_1 \approx$ 10-30 m and a magnetic Reynolds number $Re_m = R_{20}v_0\sigma_0\mu_0 \approx (3\text{-}30)\cdot 10^3$ gives much lower energy release than the case of an ideally conducting liner. In particular, for the variant with the copper liner presented above the difference in the energy release is about 40% for $R_{10}/x_1 = 14$ and Re_m = 26,200.

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